ADDRESSES

Delivered Before The

California Teachers Association

AT RIVERSIDE

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DELIVERED BEFORE THE

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BY

PROFESSORS IN THE UNIVERSITY OF CALIFORNIA.

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EDUCATIONAL PROGRESS IN CALIFORNIA.

By Professor Martin Kellogg,
Acting President of the University of California.

California was admitted into the Union in 1850. Its preliminary Constitutional Convention was held a year earlier. The years of its State life may be roughly divided into decades, of which only four are now complete. This division of time will afford a convenient method of grouping the events of our educational history.

I. We go back, first, to the beginnings of our school system. The foundation for this system was laid in 1849, in the Constitutional Convention at Monterey. An inviolable school fund was then established. Article IX of the Constitution said: "The Legislature shall encourage, by all suitable means, the promotion of intellectual, scientific, moral, and agricultural improvement." Section 4 of the same article directs the Legislature to care for a permanent fund for the support of a university "for the promotion of literature, the arts and sciences."

But there were schools antedating 1849. As early as October, 1847, the Town Council of San Francisco provided for the erection of a small school-house, and in April, 1848, it was occupied, with Thomas Douglass (a Yale graduate) as teacher (salary $1,000 a year). There were at first six pupils; the number increased to thirty-seven, then suddenly dropped to eight. The

dropping off was due to the mining excitement which swept so many off to the interior, and among them teacher Douglass.

During these years, private schools were opened by Messrs. Marston, Williams, and Pelton.

The first school law was passed by the Legislature of 1850–51. Any district drawing money from the school fund must maintain a school at least three months in the year. This law provided for the establishment of high schools. It allowed religious schools to receive aid from the fund.

San Francisco had had, in 1850, a free public school, independent of any State law. The Common Council established it, and John C. Pelton was at its head.

According to the report of the first State Superintendent, in 1851, there were about six thousand children in California between the ages of 4 and 18.

II. Let us now pass on to the neighborhood of the sixties. In 1858 there were over forty thousand children of school age, of whom less than one half attended the public schools. In 1855, religious schools had been shut out from the school fund. In 1861, an attempt to admit them again to its benefits was defeated. Much interest, during this first decade, had been shown in developing the school system. The patrons of the schools still paid rate-bills. The schools were not yet in a condition at all satisfactory to the State Superintendents, as is evinced by their reports.

I turn now to another line of development, viz.: the beginnings of college instruction. If I give this a large place, I plead in excuse the "personal equation," as my environment has given me a special interest in college work.

In 1860, the College of California began college instruction in Oakland. It was not the first college on this coast. Other colleges, or "universities," were earlier in the field. But those other institutions depended chiefly on what we call preparatory classes. Whoever aspired to a college degree must take his advanced studies as he could get them, in company with scholars of lower grade. The one noteworthy distinction of the College of California was this: It had no preparatory school work to look after; it had a Faculty exclusively devoted to college studies. Its standard was the standard of the Eastern States. This made a marked advance upon any previous effort.

From the first years of California's statehood, a few educated men had ooked forward to such a college. In 1849, a charter was sought through the first Legislature. The bill authorizing such charters was passed, leaving the special application to be made to the Supreme Court. A financial basis of $20,000 was to be a prerequisite. A cluster of men, afterward interested in the College of California, had obtained a promise of certain tracts of land and applied for a charter, but defective titles defeated the application. This was in 1850.

In 1853, Henry Durant came to do educational work in California, and started a preparatory school in Oakland. He interested a few college men in his enterprise, which, from the outset, aimed at the building up of a college. A small building was rented at a high price, and his school began with three pupils. In 1855 the three had increased to sixty.

The same year a charter for a college was obtained, the Trustees named being Fred. Billings, Sherman Day, S. H. Willey, T. D. Hunt, M. Brumagim, E. B. Wals-

worth, J. A. Benton, E. McLean, H. Durant, F. W. Page, R. Simson, A. H. Wilder, and S. B. Bell.

These Trustees had very definite aims. One of these was to unite the religious people of the State in an effort to establish a first-class college, which, like Yale and Princeton, should be Christian in tone, but practically unsectarian. Starr King was a member of the Board, and after him Horatio Stebbins. The very breadth of their scheme increased the difficulties of the Trustees. Devoted denominationalists could not be deeply interested, and waited for an institution all their own.

Another definite aim was to place the college on a high footing by obtaining for a President some distinguished man from the East. Dr. Horace Bushnell was almost secured. He was one of the loftiest souls in the whole land, and was deeply interested in education. He came to this State for his health, and spent much time in looking up a good college site. With health regained, he yielded to the importunities of his old church, and was lost to California. Drs. Shedd and R. D. Hitchcock in turn declined the offers of the Trustees.

Thrown back upon their own efforts, the Board moved forward. They secured a college site, now occupied by the University at Berkeley, and dedicated it on April 16, 1860. In the summer of that year the college began its work in Oakland. Its first Freshman Class had been trained by Dr. Durant. The first professors chosen were Durant and Kellogg, the latter of whom began work in 1861. Prof. Brayton took certain classes, but gave most of his energies—too unsparingly—to the school left by Dr. Durant.

Thus, after the first ten or twelve years of California's

life, we see the full establishment of a common school system, and the beginning of separate, determined college work.

III. We pass on to the seventies, to the close of a second decade. What advances have these years to show?

On the part of the schools, a great expansion and a higher level. Early in the sixties, increased provision was made for a school fund. John Swett became State Superintendent, and argued strongly for a special State tax. Such a tax was laid in 1864. The year ending with June, 1867, "marks the transition period of California from rate-bill common schools to an American free school system." The Superintendent records this fact with pardonable pride. A school library system was provided by the law of 1866, and put into successful operation. The average salary of male teachers was reported as $77 a month; of female teachers, $64. Fifty thousand children were enrolled in the schools. From 1861 to 1871 there were eight State Institutes. At the third of these, called in San Francisco by Superintendent Swett, there was an attendance of four hundred and sixty-three. In 1867, five hundred teachers were present. These Institutes were of much value in giving the teachers an *esprit de corps*, and in influencing school legislation. In 1869, the State tax was increased to 10 cents on each $100.

On the side of the higher education there was an advance from the college to the university. As in the case of the college, the university idea had long been cherished: how it should take shape, was a question to be determined by events. As has been mentioned, the Constitution of 1849 made reference to a

university, and enjoined on the Legislature the protection of funds granted for such an institution. In 1856, Superintendent Hubbs urged the establishment of a university. His successor, Superintendent Moulder, discussed the subject at large.

In 1853, Congress granted to this State seventy-twc sections of land "for the use of a seminary of learning." Also, ten sections of land for a public building fund. In 1862, by the Act known as the Morrill Act, Congress gave to the State one hundred and fifty thousand acres of public land for an Agricultural and Mechanic Arts College. This donation was accepted in 1864. In the previous year the Legislature appointed a Commission to report a plan for a university, the chairman being Prof. J. D. Whitney. The scheme elaborated by this Commission was very different from the plan finally adopted. In 1866 a Board of Directors was established for an "Agricultural, Mining, and Mechanic Arts College." The next year the Directors located the college in Alameda County.

In the same year, 1867, came a proposition from the College of California, looking to a larger and broader institution, a real university. The Trustees of the college had tried faithfully to expand their own institution; but the process was slow, and the financial prospects unfavorable for the early fulfillment of their hopes. They had a fine property in Oakland, and the magnificent site of one hundred and sixty acres at Berkeley. They had established a high college standard. Through their seven years of actual college work at Oakland, and through the very successful Alumni gatherings of men from Eastern institutions, a wider interest in college instruction had been developed. The Trustees now offered to turn over their property

and good-will to a State University, with the condition that the University should maintain certain colleges of science "and an academical college," "all of the same grade and with courses of instruction equal to those of Eastern colleges." This offer was accepted by the Board of Directors of the Agricultural College. An act to create and organize the University of California was introduced in the lower house of the Legislature March 5, 1868, by John W. Dwinelle, and approved March 23d. We call that our Charter Day.

The College of California continued its instruction till the summer of 1869, and then became merged into the University. The first appointees to professorships in the University were Dr. John Le Conte and Prof. Kellogg. Dr. Joseph Le Conte and others soon followed. The first University Class entered in 1869. In 1870 there were two classes, and young women had already been admitted on an equal footing with young men.

The end of this second decade, therefore, is marked by the complete establishment of a *free* public school system; and by the establishment of a State University, taking up and much enlarging the previous work of the college. New attention was given to the claims of the natural sciences, to the demand for training for the scientific professions, and to the bearing of education on industrial occupations. Military training, the only condition imposed with the national grant, became a part of the students' education. Our State took a step in advance of any other in making tuition in its University absolutely free.

IV. The third decade brings us to the eighties.

In these years the public school system gained in

efficiency. In his report for 1875, State Superintendent Bolander notes an important advance in furnishing aid to the smaller school districts. Some of them had fallen short of the minimum of three months: now the distribution was more equal and helpful. He argues in this report for a special and extended course in the University for teachers.

During this decade there was a local political cyclone, which for awhile overturned the calculations of the old parties. It brought about, in 1879, a new Constitutional Convention, whose proceedings interest us in one or two points. First, high schools were left out of the schools to be supported by the State tax. Any community wishing for a high school must itself meet the expense. This omission of the high school, in the series of schools provided by the State, has had a chilling effect on the higher education. Common schools were free, the University was free; but how were pupils to get from the one to the other? The larger cities bridged the chasm without much hesitation by establishing high schools at their own expense. The smaller towns and country districts were slow to assume the burden.

Notwithstanding these difficulties, the University maintained its high standard of admission. It could not lower the requirements without taking a long and fatal step backward.

That Constitutional Convention took another step which was not adverse, but favorable to the State University. Article IX, Section 9, declared: "The University of California shall constitute a public trust, and its organization and government shall be perpetually continued in the form and character prescribed by the organic act creating the same." In that radical Con-

vention it was decided that the University was something not to be uprooted.

V. And now we come to the nineties.

The common schools have prospered as never before. More really well-educated teachers have been found in the profession. As a profession, teaching has become a more honorable and a more permanent occupation. We have now three normal schools instead of one. Teachers' Institutes have been made a part of the school system, and they have been attended with increasing interest, even enthusiasm. The whole machinery of the school system has been elaborated and made more efficient. The teachers have shown increasing devotion to their work, and the best of them have found appreciation in the community at large. The one weak point in the system, showing itself chiefly in the larger cities, is the method of choosing Boards of Education.

The unfortunate chasm between the common school and the University has been more and more regretted, and there have been strenuous efforts to bridge it over. Years ago the schools were authorized to add somewhat to their courses, in the direction of the scientific requirements of the University. The last Legislature passed two bills for the establishment of high schools: the one for county high schools, the other for city or town high schools and union district high schools. The burden of maintenance was still to be local, but provision was made for putting the question before the people, and for levying the needful tax. These laws have spurred up a considerable number of communities, and the work of establishing high schools is now going forward with the most favorable prospects.

The standard aimed at in these high schools is most encouraging. As one object in establishing them was to pave the way to the University, the requirements of the University have been cheerfully met. These requirements do not hinder, but rather help, in fixing a curriculum of most value to students who take no further course. It is always borne in mind that only a minority are to take a college course, but this minority must not be debarred from going forward.

A special feature of the last decade has been the attainment of a uniform standard. This has been brought about by a system of accrediting. Our University was not the first to devise this system: Michigan was already using it. But we may claim to have made the system more thorough than we found it. More pains is taken here than anywhere else to visit, advise, and encourage each one of the main departments of instruction in every school applying for recognition. The results have been very satisfactory, and the circle of accredited schools is constantly widening. Nearly thirty are already on the list.

Another gain is seen in the call for a longer curriculum in the high schools. Californians have been fond of short cuts and the briefest possible courses of study. But the State is less in a hurry than it was. It is coming to see that haste makes waste, that the basis of a broad and solid scholarship cannot be laid by a feverish urgency. A few high schools have already provided a four years' course. If this can become general, it will react on the grammar schools, and bring about in them a reduction of work and an earlier advance to the high school point. Thus those who are on their way to the college will lose no time; but they will have changed some of the needless repetitions or premature exactions

of the grammar years for the more careful and thorough studies of the high school.

The State University has made good progress during this last decade, notwithstanding the difficulties as to preparation. Some academies and private schools have helped to fill the place of the missing high schools. Liberal aid has been given by the State to its University by a continuing tax. The staff of instruction has been much enlarged, with a corresponding expansion of the courses offered to the students. Special aid has come to the Agricultural and Mechanic Arts courses from a supplementary Morrill Aid Fund. The professional schools connected with the University have a much larger number on their rolls. Denominational colleges and schools have sprung up at various points in the State.

So much for the history of this fourth decade. The fifth is but beginning. But

VI. The new decade opens with extraordinary promise of educational development. Three things are especially worthy of note:

1. The springing into life of a new and powerful university. I need not say where it is situated, nor what name it bears. The people of California have all heard of it, and have all been interested in its brilliant beginnings. To our educational force in this State an entire new Faculty has been added by the magic wand of wealth. This corps of able scholars and accomplished teachers has become known already in all our educational circles. You welcome some of their number at this meeting of the State Association.

2. A second important event of the year 1891 is the movement for the University Extension. It is already

general, and promises to become almost overpowering. It is not endemic, but epidemic, affecting the whole country, and coming almost as swiftly as the *grippe*. Earlier and more slowly it grew up in Great Britain.

But no such movement is capriciously causeless. In our own country there was a deliberate and widespread effort to popularize knowledge, through the Chautauqua movement. That movement reached a multitude of earnest minds, and has made itself felt in this State. When a higher plan came in, under the name of University Extension, it found the public ready and eager to welcome it.

Just what form University Extension is to take in California, it is too early to judge. If it can be kept on a true university level, it will do much to carry the higher instruction to those who cannot attend college classes. In that case, it will call for a large increase of the University Faculties. The field work will rival in its demands the proper home work of the universities. If too many places are clamorous for their share of attention, there will be encouragement for some adventurers, coming in by doubtful doors, who will not satisfy the more thoughtful minds. There are many problems still unsettled in the application of this system to our communities. But some form or forms of UniversityExtension will surely play a prominent part in California in this last decade of the nineteenth century.

3. There is much more demand for distinctive courses of instruction in the interest of the teaching profession. Normal schools there have been for many years. Now there is a call for a similar and higher professional training in the Universities. Chairs of pedagogy have been established in many institutions. Harvard has a

new Professor of the History and the Art of Teaching. Yale gives several courses in pedagogy. The universities of this State recognize this as an important department of instruction.

This review of educational progress in California is necessarily very incomplete; but it shows that no cataclysm has taken place in our educational history. During all these decades there have been forces at work for the promotion of the educational interests of the State. If these interests are now prosperous, it is in sequence of previous long-continued and earnest efforts. If there is a harvest for the teachers of California to reap in the near future, it is because some patient husbandmen have sown the seed.

We that are older almost envy the young teachers who are to take part in this new and larger progress. But there is a certain satisfaction in having helped to lay foundations. And let me utter a note of warning. If in these coming years the teachers lose their love for their work, if they forget the self-sacrifice which that work implies, and cease to emulate the single-hearted devotion of the pioneers, if they seek rather for easy and comfortable places, then true educational progress will cease. Young ladies and gentlemen, you, too, must be pioneers for a still better and brighter future. You, too, must help to lay foundations which shall be out of sight and forgotten, it may be, that the future temple of education may rise, sacred and glorious, in this home of our love and our hopes.

THE SOCIAL SCIENCES AS AIDS IN TEACHING HISTORY.

By Bernard Moses,
Professor of History and Political Economy.

During the few moments in which I may engage your attention, I shall invite you to consider the social sciences as aids in teaching history. A certain justification for bringing this matter before you may be found in the position which history and kindred subjects now occupy in the general scheme of academic studies. What this position is may be seen by glancing at the historical development of the curriculum.

Considered with reference to its growth, the primary phase of the academic curriculum is that in which religion is the sole topic embraced. The Mohammedans, who found the Alexandrian library superfluous, in so far as its books agreed with the Koran, and useless, in so far as they differed from it, were the products and the advocates of this simple curriculum. It appeared again in the Middle Ages, when the Christian church, in its pretensions to omniscience and absolute authority, essayed to fix the limits of human inquiry. Galileo might not teach that the earth moves, because this proposition was not in the predetermined circle of knowledge.

The first considerable extension of the realm of knowledge was made through that great intellectual

movement which we call the revival of learning. This opened to modern minds a buried and almost forgotten civilization, a civilization whose art and poetry and philosophy transcended the highest conceptions of those to whom it was revealed. The study of antiquity drew to itself a constantly increasing number of scholars, and, in their enthusiasm for ancient learning, all other subjects remained in comparative neglect. But on this study of ancient literature and ancient institutions, the church looked with suspicion. It feared the anti-christian influence of pagan thought. It feared a revival, not only of the learning, but also of the worship of paganism. It feared the loss of stability and power in the presence of the unshackled mind, and, therefore, the means for developing the new learning had to grow up outside of the cloister and find other supporters than the clergy. The secular authorities in states and cities then became the patrons of learning and the founders of schools. The oldest universities of Italy and Germany were established under the impulse of this spirit; and thus the demand for instruction in the new field, which the church would not satisfy, was met by the wise liberality of princes acting in behalf of the states whose secular power they wielded. With the development of the new learning, theological teaching came out of the cloisters and was established at the universities. The church ceased to be the sole patron of education, and scholasticism no longer covered the whole field of thought and inquiry. A new topic was added to the curriculum of human knowledge. To the study of divinity was added the study of the humanities.

Through the influence of the attractive revelations of antiquity and the powerful prejudice of the church,

the early discoveries in the realm of nature either suffered neglect or were regarded as unfit material for academic teaching. But in the first half of this century was revealed the second great intellectual revival of modern times. The fragmentary and unsystematic investigations of earlier scientists grew into a systematic and well-ordered search for scientific truth, and the results of this search, not less wonderful to the nineteenth century than were the revelations of the literary revival to the fifteenth, have grown into that body of knowledge which we call modern science; and this science, opposed at first by the church and neglected by the humanists, now takes its place by the side of scholasticism and the humanities in the great universities of the world, and forms a new addition to the field of learning. Thus to the primitive curriculum embracing the subjects properly involved in the study of religion, each of the ages in question has added a new topic or a new department of academic instruction. One has added the humanities; the other, modern science.

In these last decades, a fourth topic, or department of study, has been added to the list, and it could not well have been added earlier. During the seventeenth century and the early part of the eighteenth, the period of European absolutism, when society was supposed to be directed arbitrarily by the capricious will of an absolute ruler, there was little incentive to study the history of society or to attempt to discover the laws of its activity and growth. But by degrees it came to be generally acknowledged that the ultimate political power in the state, that power which is the source of all delegated authority, rests with the people. So long as the people had no share in the political control

of society, their knowledge or their ignorance of the principles and practices of public administration was a matter of little concern. There was no inducement for them to acquire knowledge of the details of governments in which they had no voice; but when the power to decide, either directly or indirectly, all questions of public interest devolved upon the people, it became a matter of grave importance that they should be instructed in the history of society and in the laws and principles which underlie its organization, activity, and growth. There arose, therefore, a demand for an enlargement of the field of instruction, which should embrace especially the subjects of history, economics, and politics; and this group of subjects constitutes the latest important addition to the curriculum of academic studies.

During the last forty years this department of knowledge has become more and more important in the academic instruction in America, and the leading universities have yielded to it a wider and still increasing place among the several courses of study. Many of us, still not very old men, remember distinctly the time when history was not considered of sufficient importance in the curriculum to demand the exclusive attention of an instructor. It was turned over somewhat indiscriminately to any member of the Faculty who might happen to have fewer and less exacting engagements than the others. It was not thought necessary to consider his attainments; any one who could read and understand a plain narrative ought to be able to teach history. What changes the period of a generation has wrought in the academic standing of these subjects may be seen by observing their present place in the curriculum of some of the leading universities

of this country: Harvard has forty courses taught by twelve instructors of different grades; Yale thirty courses, with ten instructors; Columbia thirty-one courses, with eleven instructors; Cornell twenty-seven courses, with ten instructors; University of Michigan thirty courses, with nine instructors; University of California twenty-two courses, with four instructors.

An examination of these courses will reveal the fact that while the several studies of the group dealing with historical, economical, and political phenomena bear different names, they are all nevertheless designed to contribute to a common end, and that is to render a rational account of society. The whole group, as now constituted, is in some sense an outgrowth of the course of instruction in history as primarily organized. Its purpose is an enlargement of the purpose of instruction in history. As formerly presented, history showed us somewhat of the *activity* of a nation, but it left much to be known as to the *nature* and *organization* of the nation. To supplement the knowledge of society which history furnished, the science of economics was introduced, which undertook to give a scientific account of the *industrial* and *commercial* organization and activity of the nation; while a little later political science came to enlarge our knowledge of man's *political* relations and activities.

All the subjects arranged in the curriculum under these three general heads of history, economics, and politics will be found, on thorough examination, to be closely united by lines of mutual dependence. History, as the comprehensive record of the internal affairs and foreign relations of nations, necessarily involves discussions of the subject-matter not only of economics, but

also of politics. The internal affairs considered are not simply those relating to distinguished persons, on whom has fallen by the accident of birth the nominal guidance of the public administration; rather that part of internal or domestic history which relates to the industrial and commercial order and progress of a nation. The popular side of national life has become especially conspicuous during the present century, for two reasons:

1. Because of the unprecedented development of the commercial and industrial interests of society.

2. Because of the transfer of political power from an irresponsible head of the state to the body of the nation.

These changes have turned attention to the people, and led the historian to emphasize especially popular interests and enterprises. These phases of national life have become so conspicuous that it has been found to be desirable to rewrite, from a new point of view, the history of certain nations, particularly those preëminent in industry and commerce. The history of England, therefore, in its latest version, under the hand of Mr. Green, becomes a "history of the English people." Another consequence of the conspicuous position to which the people have risen in this century is the development of studies in civilization, whether in the form of the German *Culturgeschichte*, or the French and English history of civilization, or studies on the nature and progress of society. The internal history of any nation now demanded from the teacher is that which includes not only the phenomena of trade and industry, of money and taxation, but also the whole body of facts from the life of the people, which constitute the data of economics, the phenomena which it is

the business of economics to explain. In order, therefore, to understand that part of a nation's history which is here referred to, one has need to know not merely the data of economics, but also the explanations of the data which the science of economics has to offer. To know history, one must first know something of economics. To be an historian, one must first be something of an economist.

In so far, moreover, as history sets forth foreign relations, it describes public activity in reference to that body of rules which are supposed to govern nations in their dealings with one another; and in order to understand this activity, and to be in a position to speak with critical judgment upon it, which it is the business of the historian and the teacher of history to do, one must know this body of rules; in other words, one must have a sufficient knowledge of international law.

The teacher who has read carefully in economics and international law will be in a condition to bring to his classes illustrations and explanations which hitherto have been generally wanting in the teaching of history, and because they have been wanting students have regarded the subject as somewhat stale, flat, and unprofitable; and in this matter the judgment of the students does not appear far from the truth. The history of the action of the French Revolutionary government regarding its revenues and expenditures, for example, has no interest, and appears to have little significance, except as we are able to bring to its consideration a knowledge of the economics of money and the fundamental principles of finance. In the history of this country, moreover, there are many phases which cannot be fully comprehended, except in the light of

economic principles; and many controversies into which we have fallen with other nations cannot be adequately appreciated and correctly estimated, except we know the bearing of international law on the questions involved.

But as an aid in understanding history, the science of politics is even more important than economics or international law. Dealing with the nature, functions, and structure of the state, it enables one to get clear and definite conceptions of the elements whose activity constitutes the subject of the historical narrative or discussion. Unless we are able to assign a precise meaning to the various terms used relating to government and society, historical writing will necessarily appear vague, and the vagueness will either become wearisome and prevent long-continued reading, or will engender indifference to accuracy of thinking. But if we know that sovereignty means the power vested somewhere in the political organism, by which the fundamental positive law is issued and may be changed, and that the sovereign is the person or organized body of persons holding this power, we are in a position to appreciate the full significance of these terms when used correctly, and to detect the error when used incorrectly; and by thus always moving in the light of a clear understanding, we may avoid confusion, and hope to reach the truth. On nearly every page of history there is some record of the action of the state, and a large number of readers are in the absurd position of following the narrative of a course of activity, without having a clear and definite conception of the limits and qualities of the subject acting.

And before the science of politics was developed for the enlightenment of the historian, his work showed a

tendency to become a royal chronicle amplified by voluminous details of military affairs. But scientific analysis of political phenomena has helped to clarify our views of society, and make history in this age more completely than ever before a comprehensive account of the progressive national life. It has revealed to us, moreover, the distinction between the state and the nation, showing the state not as a divine incomprehensible something, but as that body of persons in the nation who have political rights, and are united in an organization to wield the power of the nation in behalf of the nation. The state, therefore, stands revealed by modern political science as a purely human institution, which has come into its present complex form only gradually as the needs of public agencies to do the gradually increasing public work have appeared.

Through the science of politics we are made to see not only what the state is, that is to say, what the organism is whose actions form a large part of the subject-matter of history, but also to see by what special agencies within the state the various departments of the public work of society are carried on. On this point the student in the early years of his study of history has much need of enlightenment; for the historian, while treating of the progress effected by the public action which has been carried on through these agencies, often leaves us entirely uninformed as to the nature, structure, and relations among themselves of these agencies. Yet, under the present organization of instruction, students may not be expected to acquire this information before beginning the study of history, however necessary it may be to clearness and accuracy of historical knowledge.

If, then, we conclude that much of the information

which is involved in the sciences of economics and politics is essential to an enlightened understanding of history, and that students cannot be required to make independent acquisitions in this line until late in their academic life, we are obliged to find means to remedy this defect and prevent their early-acquired historical knowledge from being vague and unprofitable; and in this search we are turned to the method of teaching.

In presenting a subject in history, no teacher in these days is likely to be satisfied to leave the topic with the meager statement of the text-book; and he is, therefore, compelled to devise some feasible means for extending this information. In this matter he will be led to accept not necessarily the best means abstractly considered, but the most serviceable in view of the time and appliances at command. These usually reduce themselves to two. The first is to require the student to read indicated passages in various authors on the topics treated in the text. Given practically unlimited time, all the books desired, and a certain ripeness of the critical judgment, this method may be employed with great advantage. But these conditions are all wanting in the schools below the university, and even in the lower classes of the university itself. Whenever a large class is being taken over a given course of instruction, and all the members are reading the same list of indicated passages, they will meet with embarrassment in any library in this country, on account of lack of books. This plan is, therefore, practicable only after the elementary stages of historical instruction have been passed, those stages in which the work of all the students is of necessity essentially the same.

Even if these physical difficulties were removed, it would not be advisable to give this method more

than a limited application. In the early days of the discovery of German universities by American students, the methods there employed made a profound impression. The young men who came back to teach in American schools or colleges, were moved to give instruction by lectures. They forgot the conspicuous differences between their own positions and the position of the German professor. They forgot that the German students came to the lectures already possessing many of the attainments which belong to a liberal education, and that the professors had something to communicate not found in the ordinary text-book; often the results of their own investigations, which had not been embodied in the literature of the subject in hand. The professor of the German university spoke to young men already familiar with the elements and outlines of the sciences, and who wished to learn the sources of further information. Pointing out and criticising these sources was often a large part of the lecturer's work. These conditions were all wanting, and are still wanting, in a large part of the field of academic instruction in this country; and the German method of lecturing, apart from its proper conditions, was absurd. It had an application and was the only practicable method in the early history of education, when books were few and inaccessible to most learners. Again, in these times, when books have become bewilderingly abundant, the academic lecture, in so far as it is a giving of special methods of investigation and criticism of sources, has become useful for students of large attainments. These conditions may be found in the graduate department of our universities, and perhaps to a certain extent in the highest grades of the undergraduate courses.

In these last years, moreover, the American teacher has discovered the seminary method. This was also brought to light in Germany. The plan of instruction which it involves has numerous excellences, but it is not adapted to all circumstances. It is undoubtedly a proper auxiliary in the highest grades of instruction where students under competent direction are pursuing independent lines of research. But its extensive employment in the early years of historical study is questionable. Specialized investigations in history from the beginning of the student's career lead to the danger of knowing everything without knowing the whole. The knowledge of the facts relating to any particular event, unsupported by a synthetic understanding of the whole course of human experience, and by an appreciation of the historic place of this event, is scarcely more important than a knowledge of the fictitious events and characters of Scott or Balzac. History as an academic study means more than this; and, to make it more, some plan of study must be had which shall defer the minute investigation of special points till the mind has been trained to grasp as a unit the whole record of progressive society. If one would be an historian, or merit mastership in this department of knowledge, it must become habitual for him to view events in the large. To this end, the scheme of western history should be unfolded before him at the beginning of his studies, and kept before him through all his subsequent work. At first, a mere skeleton in his mind, from year to year it acquires a greater and greater fullness of detail, until at last he finds himself in possession of a comprehensive and well-balanced general knowledge of history. At this point some limited field may be selected, and made the

object of his special inquiries. To this he may then direct whatever force and scholarly insight he can command, and not be in danger of failing to appreciate the proper relation of his special subject to the whole of which it is a part. But to direct the immature student to the study of special points in history is to cause him to become habituated to small views, and to render him liable to be hopelessly lost in details. He may know the record of certain events, but never rise to the larger conception of history.

The second plan for extending the range of the student's information is that under which the teacher furnishes a more or less elaborate commentary on the points involved in the text. In carrying out this plan, an item of first importance is to determine what books shall be used by the students. An objection is raised against the use of the ordinary text-book, on the ground that it is meager and lifeless, and that supplementary reading cannot be done by a large class with advantage. If there were no physical difficulties in the way, students would be likely to read extensively on some points, and only a little on other points, perhaps more important, according to the facility of obtaining the literature desired. Both of these difficulties may be avoided by putting into the hands of each student a book by a recognized master of the subject to be considered, and not by a mere text-book maker, which shall contain all that we may require the student to read, even when under the highest pressure. If, to illustrate by a single example, the subject for the term in the university is English history in the eighteenth century, let each student have the eight volumes of Lecky's "History of England in the Eighteenth Century." The advantages of this over the small text-book and outside reading

are numerous. In the first place, what the student reads on the subject will be in proportion to the importance of the several parts of it, since he will take it in the form in which a great historian has viewed it and set it forth. He will, moreover, have all the material of his work at hand for reference at all times. He will learn to read a voluminous book, which is a matter of vast importance for the youth who would be a student; and after a few undertakings of this kind, he will not be deterred from seeking information because it happens to be contained in a work of several volumes. The difficulties which arise from attempts to supplement the deficiencies of the small text-book do not appear here; and if the opposite may sometimes hold, the teacher may use his discretion in directing limitations, and no embarrassment will ensue. With such a text before the student, the teacher will not be tempted to waste time in giving, by way of informal lectures, what can be much better acquired from the book itself. Nevertheless there should be comment, but the nature of it will depend very largely on the character of the book used; but in general it may be said that the teacher should illustrate by setting forth the principles of social order and progress, rather than consume the time in elaborating the narrative. Merely extending the story of historical events adds very little to our understanding of the subject; but analysis of the state and society reveals to us the mode of organization, the instrumentalities of social action, and the forces by which the nation is moved along the course of improvement.

But this involves on the part of the teacher certain attainments in economics and political science, and it is the importance of these attainments for the teacher of history that I wish to emphasize. The special char-

acter which historical studies will assume under different circumstances will depend very much on the kind of opportunities which these different circumstances present. In European countries where the scholar has access to the sources of history, whether in great libraries or in the archives, his investigations are likely to be largely directed to setting forth hitherto unknown details. But in a country like this, whose civilization is but of yesterday, and which has no sources except for the history of a few recent generations, historical study will necessarily be turned to some other end. The American scholar is unable to compete successfully with the European scholar in searching out the details of early European history; and his energy and taste for social studies will tend, therefore, to find satisfaction in the scientific interpretation of the historical facts presented by European historians, and in the development of those social sciences which take their data from history.

But hitherto the achievements of this nation on any of these lines have not been remarkable, yet this fact does not indicate a lack of worthy political thought. In the field of political theory, Roman literature, to illustrate, is specially barren, yet no one will deny that the Romans thought much and well on political problems. Their thinking was given expression and form in laws and practical institutions rather than in treatises; and in this regard the people of the United States are comparable with the Romans. For more than a hundred years this nation has been deeply absorbed in the work of building a great state; and the best the nation has thought has been embodied in the institutions and laws, giving them in some sense the character of original creations. But the large place occupied

by social subjects in the universities, and the large number of men giving to these subjects their thoughtful attention, are conditions specially favorable to the production of an important economic and political literature. Here the parallel with Rome ends. While the Roman republic fell under imperial rule which choked all political discussion, this republic has entered upon that phase of its history in which thoughtful political discussion has become a necessary condition of continued healthy existence. And the present activity in the study of society suggests that the nation may yet make achievements in economics and political science worthy of the originality already displayed in practical politics.

There are good reasons why some part in cultivating this field should fall to the teachers of history, not merely in the universities but also in the other schools throughout the country. The first of these is the general fact that independent work, which results in an advancement of knowledge, is essential to the highest efficiency of the teacher. The teacher who ceases to be an investigator, who ceases to seek to enlarge his mental vision by scholarly inquiries, is in danger of stagnation and the consequent loss of the reasonable ground of his calling. Another circumstance by which the already mentioned line of study is commended as a proper field for the teacher's independent inquiries, is the fact that, while in the realm of pure historical research adequate sources of information are found in only a few places, in the auxiliary subjects of economics and political science one may always have at hand abundant material for original work. There are always within his horizon the data of some hitherto unsolved social problem. The society in which he lives is the

laboratory in which experiments are constantly going on before his eyes.

The analysis of social institutions, moreover, will facilitate an understanding of that peculiar and distinctive characteristic of western society which we call progress, and thus throw a light on history that cannot be derived from any other source. In this light one may see clearly the unsoundness of the favorite expression of the half-instructed, that history repeats itself. With our knowledge of history, enlightened by an understanding of the structure and moving forces of society, it becomes evident that that movement which is observed in progressive society is never in a circle, but always on to new ground, though not always in an unswerving line forward. Progress is not continuous, nor always at the same rate; there are periods of halting, and even of retrogression, when deterioration marks many of the phases of life, when a mortal disease appears to have smitten a whole civilization. It perishes like the seed that is sown, but after the destruction there rises a new and more fruitful life. The growth of society is, moreover, like the growth of a great city; built up little by little, it serves the purposes of a generation, but at last fails to meet the new needs. One by one its buildings are replaced by others of better plan and sounder make. Thus in social growth, old institutions that have fulfilled their mission yield in time to institutions suited to the new wants of a more advanced society. A great revolutionary movement sometimes sweeps the institutions of a nation to destruction in a day, as a fire sweeps away the various structures that have served the diverse purposes of civilized existence. Then the work of construction is rapid and free; and as the new city is

better than the old, so are the new institutions adapted to a higher and better social existence. But the old structures are never rebuilt, nor the old institutions recalled into active force.

This general truth finds another expression in the statement of the fact that the practical social problems of no two ages are ever exactly the same. If the ages are far apart, all the conditions, even the nature of the persons involved, may be different. The men of this age, for example, who know from observation the marvels of modern life, have, as the creatures of other circumstances and other traditions, a nature different from that of the men who lived the narrow human existence of the Middle Ages. They are distinguished by a wider range of intellectual vision, a more comprehensive grasp of the imagination, and a series of new impulses. Moreover, the physical conditions of to-day are utterly at variance with those of earlier centuries; and these two facts alone are adequate to make the social problems which we have to solve new problems. Because the problems of each age are new, social advancement can be achieved only by slow and costly experiments. The general place of experiment in the progress of society the teacher of history should understand, and for enlightenment he must turn to those writers whose business it is to analyze society and to set forth the laws of social order and growth.

The fact that some points are still in debate in the sciences which deal with the phenomena of society, is not an argument against their utility as subjects of study. Every science has a margin of unsettled questions. These constitute the principal attraction for the independent investigator, and furnish him an opportunity to make a permanent contribution to human

knowledge. The comparatively late development of the social sciences is not wholly due to the influence of absolute government practically prohibiting the discussion of questions relating to political affairs, but also in part to the complexity of the data involved. The attempts to render a scientific account of society have made unusually prominent certain social phenomena, which, since they have been made specially conspicuous, the teacher of history is bound to consider. Even if he would, he may not present history as a chronicle of the achievements of the distinguished head of the state. He must set it forth in its character as the record of the most complex of organisms; and to learn this character there is no better source than the writings of those by whom this complexity has been specially revealed, and through whom a certain degree of scientific order has been brought out of the chaos of social phenomena.

But political science has encountered even more hinderances than economics. The claims of monarchical rulers to hold a power of divine origin, and to be free from responsibility to the governed, made the theory of politics in some sense an appendage of theology. From the conception of the monarch as the agent of God, and of his power as the supreme power in the state, the step was easy to the conclusion that the state itself was divine. Metaphysics applied to politics became sterile and unprofitable. The only possibility of progress lay in taking a new point of view and a new method. This was not readily accomplished, for men hesitated to encounter the thunders of the metaphysical theologians, or the theological metaphysicians, which would necessarily follow the declaration that their whole philosophy of the state was unreal and empty.

Nevertheless, the interests of progressive knowledge demanded that the assumptions of metaphysical politics should be abandoned, and a new start made from observed and known data. With this beginning, and using the methods of ordinary scientific research, important steps have already been taken toward the creation of a science of politics that will explain the phenomena of which it treats. And it is to this, and not to the theologico-metaphysical politics, that the teacher must look for light on the political and constitutional phases of history.

An important function of the teacher of history, therefore, under the method suggested for work below the highest, is that of a judicious commentator on the topics involved in the student's reading, and in making these comments no information will serve his purposes better than that drawn from economic and political science, in which are set forth the laws of social organization as well as the forces and methods of progress.

THE PAST AND PRESENT OF ELEMENTARY MATHEMATICS.

By Irving Stringham,
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In the long history of the world's intellectual progress nothing is more striking than the dependence of each succeeding age upon its predecessors for the materials out of which its own achievements are wrought. The raw materials we use in the manufacture of science are seldom dug up fresh from the earth, but are picked up by the wayside, where they were dropped by our predecessors, who could devise no use for them. It thus happens that the discoveries of Newton are old hints wrought over and made into the finished product of science by the master-mind, some of them dating backwards nineteen centuries, to the time of Archimedes. The most transcendant genius works under limitations, cannot see far into the future; and if, by a fortuitous inspiration he anticipates a discovery that, standing out of relation to the knowledge of his own time, belongs to a succeeding age, it remains unused, unfruitful, and of no current value till such time as it can be correlated with other scientific knowledge.

And so every science has a prehistoric stage, in which principles prospectively important are at first merely the incidents in apparently unrelated problems or investigations, their first users perhaps entirely unknown and never to be identified. Only later,

possibly after repeated recurrence in investigations of which it is the true foundation, does the principle call attention to itself as of great importance. It is the recognition of its importance, not its first merely incidental use, that constitutes real discovery.

The prehistoric period of mathematics belongs to the centuries immediately preceding the earliest development of Greek philosophy, and appears to have been first cultivated in connection with land surveying and astronomy in Egypt and Assyria. It had its beginnings as a science in the latter part of the seventh century, B.C., at Miletus, where the Greek philosopher Thales, who had traveled extensively in foreign countries and had resided for a time in Egypt, first taught geometry deductively; but nearly contemporaneously also at Crotona, where Pythagoras first established the principles of the doctrine of proportion, and laid the foundations of a goodly part of elementary geometry. Thenceforward, so long as Greek civilization existed anywhere, the study of geometry never ceased to be an important factor in the world's intellectual progress, and for more than twelve centuries the continuity of its study was unbroken. First, Thales at Miletus, then Pythagoras at Crotona, then Hippocrates at Athens, then Euclid at Alexandria, then Archimedes at Syracuse, then Apollonius at Perga, then Pappus at Alexandria, wrought out in succession, classified and organized into scientific unity the great mass of propositions which constitute the subject-matter of geometrical study in our schools to-day.

Near the close of the Greek period, during which geometry, from the merest beginnings, had grown into a comprehensive system, the first published accounts of an algebraic analysis appear, the Arithmetica of

Diophantos, the last contribution of the great Alexandrian school to mathematical science. This work, remarkable for its achievement of results, but lamentably defective in method and organic unity, had in it the foreshadowings of a new science, but was in no respect the science itself. It possessed no power of further growth from within, and when algebra finally became scientific it was constructed on entirely new models.

But even if the work of Diophantos had been constructed upon correct scientific principles, it would still have passed speedily into oblivion, for the light of Greek civilization was already feeble and flickering and was soon completely extinguished by the fanaticism and bigotry that for seven centuries enveloped Europe in almost total darkness. In fact, the work of Diophantos was not rediscovered in Europe until the middle of the sixteenth century.

With the disappearance of the Greek schools there seemed to be no hope for the further cultivation of mathematics as a science on the face of the earth, for nowhere else in the ancient world, up to that time, had any results of a high order been achieved. But at this critical juncture a new light appears for us in the east. The Arithmetica of Diophantos had been published (conjecturally) in the fourth century of our era, and the Alexandrian school continued in existence until the Mohammedan conquest in 641 A. D., during which time mathematics was still cultivated, though feebly, in the form of commentary or perfunctory study, and without originality or fruitful result; and it was more than a century and a half before the final catastrophe—the capture of Alexandria and the burning of the great library by the Mohammedans—that there appeared in

India a work on algebra and trigonometry by the astronomer Aryabhatta, of which no Greek mathematician of the earlier centuries would have been capable. From this time until the revival of learning in western Europe the Indians are the true discoverers in mathematical science. To what extent they were indebted to the Greeks for the raw materials out of which they constructed their algebra is not known, nor is it of consequence, for since the Greeks never succeeded in constructing an organic system of algebra, the indebtedness could, in any event, have been but small.

To the Indians, then, is due the credit of first creating algebra as a science. Two great works on mathematics and astronomy attest this claim; one by Brahmagupta, written in 628, which expounds a complete system of algebraic analysis, the other by Bhaskara, in 1150, upon arithmetic and algebra, in which the Indian system of arithmetic—the one we ourselves use—is employed. These works, however, record the highest achievements of the Indians in mathematics, and all subsequent progress in algebra has taken place in the west, and in modern times.

Though the Indians had shown themselves to be consummate algebraists, they were in no sense geometers; and far from adding any new thought to the science which the Greeks had created, they apprehended with difficulty and with much blundering what of geometry they received from Greek sources. Thus two great civilizations had stood on either side of a barrier which neither could pass. The Greeks had created geometry, but they could not invent algebra; the Indians invented algebra, but they could not add one proposition of importance to geometry.

Out of these two independent sources issued the two

distinct streams of mathematical thought that flowed first sluggishly through the Mohammedan countries of Arabia, Africa, and Spain, thence finally into Christian Europe, where they were subsequently joined together. For it was from the Arabs that mediæval Europe first acquired in the twelfth century some knowledge of both algebra and geometry. The Arabs, however, were not good conservers, or compilers, of the scientific knowledge accessible to them from Greek and Indian sources, and transmitted it in such imperfect form that many important principles had either to be rediscovered, or sought for in the Greek or Sanskrit, before Europe acquired full possession of the mathematical knowledge which Greece on the one hand and India on the other had contained. But be this as it may, Europe rapidly recovered, during the four centuries from the twelfth to the sixteenth, substantially all the mathematical knowledge that the ancient civilizations had bequeathed to their successors.

During the sixteenth century mathematics received its share of that activity in intellectual pursuits which has received the name of the Renaissance; but its development was strictly upon the lines that had been marked out for it in ancient times. Geometry did not free itself from the limitations as to method and scope that had been set for it by Euclid, Archimedes, and Apollonius. Algebra followed the Indian model, which had been recovered with much difficulty by the help of the Arabs, and, barring a few geometrical constructions of algebraic equations, mere translations into modern forms of expression of well-known Greek demonstrations, did not get beyond the limited definition of algebraic quantity as rational or commensurable number. Algebra and geometry were still two distinct

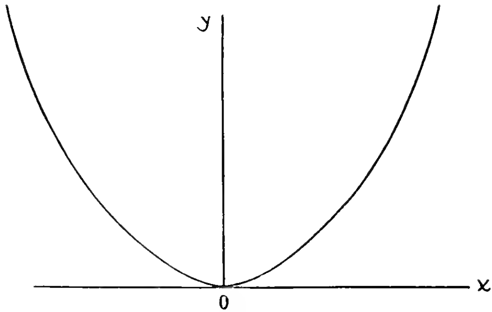
sciences, and there were no indications, except in a few special instances, that the one was in any way susceptible of interpretation in terms of the other.

In the seventeenth century, however, three important steps were taken towards a recognition of the intimate relations that we now know these two main divisions of elementary mathematics bear to each other, and which play such an important part in all modern mathematical investigations. The first was through the invention of logarithms, in 1614, by Napier; the second through the invention of analytic geometry, in 1637, by Descartes; the third through the invention of the differential calculus, in 1665, by Newton.

The introduction of logarithms, however, important as it was as an essential part of the new analysis soon to be created, was recognized at this time only as simplifying the processes of arithmetical calculation.

Both the analytic geometry and the differential calculus seemed to be necessary prerequisites for a full understanding of logarithms as a part of analysis, and a century passed before their importance as such was recognized; furnishing, in fact, a remarkable instance of a premature discovery. The ability to recognize the true import of the logarithm was made possible only by discoveries that came half a century after the logarithm itself had been invented.

The discovery made known to the world by Descartes, in 1637, may be summed up in the words: Every function has a graph. To illustrate by a very simple example, the function x^2 has for its graph a parabola with its principal vertex at the origin of coördinates, and its principal diameter coincident with the y-axis. Thus:



The values of x are represented by lines drawn horizontally from the origin O, to the left or right, from the extremities of which lines are drawn upwards to represent the values of x^2. Each pair of lines thus determined by a pair of values x, x^2, fixes a point in the plane, and the aggregate of all such points range themselves along the curve known as the parabola. The second quantity x^2, it is now customary to represent by a second letter, as y, and to speak of x and y as functions of each other. In a similar manner any algebraic equation between two quantities, x and y, has its graph.

Now mark the radical departure here taken in the interpretation of algebraic quantities. A straight line stands as the representative of any such quantity, an interpretation wholly repugnant to Greek geometry. So long as his practice conformed to the canon which, from time immemorial, had been his guide in the geometrical interpretation of quantity, x^2 could mean for the Greek only a square, an area, never a straight line. But his orthodoxy was the barrier to his further progress, and from the abandonment of that orthodoxy the modern mathematician dates the possibility of achievement beyond the limits of investigation which the Greeks had set for themselves. Henceforth, more-

over, the interests of algebra and geometry were one and the same; progress in the one was to mean a simultaneous progress in the other.

Twenty-eight years pass, and we stand at the threshold of the greatest of the discoveries in mathematics of modern times—that of the differential calculus. The year is 1665, when Sir Isaac Newton communicated to some of his friends the fundamental ideas of the new method.

Taken in connection with the new interpretation of the algebraic equation by Descartes, the scope of this method was so great as eventually—that is, within another century at most—to reconstitute the entire body of mathematical science upon a new basis, and to completely change the attitude of mathematicians towards the problem of its further advancement. Henceforth its various parts are not looked upon as disassociated systems having no common meeting ground, but geometry, algebra, trigonometry, analytic geometry, the differential and integral calculus, are seen to constitute one organic whole, or rather one organic unit in a much larger whole. Henceforth mathematics thus newly constituted is to have its alphabet, its nomenclature, its language, and whoever would use it for any purpose must learn its nomenclature and speak its language. Henceforth it is not to be the plaything of the philosopher, nor the recreative and disciplinary study of the scholiast, but the instrument of research and of efficient accomplishment of practical ends in the hands of the astronomer, the physicist, and the engineer. Such was the unparalleled achievement of the seventeenth century.

But I must be more explicit. I am endeavoring to trace out, in brief outline, from the earliest times to the

present day, the development of the foundation principles of the two great divisions of elementary mathematics, geometry and algebra (but in particular and chiefly the latter), and that which concerns us primarily in the work of Newton is his introduction of the continuous variable as one of the elements of algebraic analysis. For our present purpose it will be sufficient to describe the continuous variable as a straight line, having one of its extremities fixed at a point O, while the point P, which marks the other extremity, is free to

move forwards or backwards in a straight line. When P moves without interruption in its path the variable quantity OP is said to change continuously, and is called a continuous variable. It may or may not be possible to represent it by a rational, that is, a commensurable number.

At last the materials for a complete grounding of algebraic science, in logically fundamental principles, were at hand. Napier had given us logarithms, Descartes had put into our hands adequate means for the graphical representation and interpretation of the algebraic function, Newton had shown us how algebraic quantity could be freed from the limitation by which its meaning had always been confined to rational number. The *materials* for the work were indeed at hand, but they were in great part *raw materials*, and the energies of mathematicians were expended upon testing and reshaping the powerful instruments of analysis just discovered and enlarging the scope of their application, and little attention was given to a reëxamination of the foundations of algebra. Yet, as a science, founded upon logically scientific principles,

algebra was still in the formative stage and required this reëxamination in the light of the new discoveries. Two centuries have been just sufficient to accomplish the task.

During the eighteenth century no important result was attained. The old algebra had acquired, during the centuries since the revival of learning, a momentum sufficient to carry its development forward upon the lines that had been originally set. But at the beginning of the present century a distinct step in advance was taken, by Argand and Gauss, through the introduction of the so-called imaginary as a quantity, having equal importance in algebra with so-called real quantity, and being susceptible of real geometrical interpretation. This step made it possible, as soon as its importance was fully understood, to define algebra for the first time.

When a series of elements operating upon each other in accordance with fixed laws produce only other elements belonging to the same series, they are said to constitute a group. Thus all positive integers, subjected only to the processes of addition and multiplication, produce only positive integers, and hence form a group.

The effect of introducing into the arithmetic of positive integers the further processes of subtraction and division is to break the integrity of the old group and form a new one whose elements include, not only positive integers, but all rational numbers, both positive and negative, integral and fractional. A final step through involution, or the extracting of roots, with its allied processes, leads in a similar way to imaginary and complex numbers—that is, numbers composed of both a real and an imaginary part.

Now if, as is legitimate, we regard all reals and imaginaries as special forms of complex quantities—reals having zero imaginary parts, imaginaries having zero real parts—then the algebraic processes of addition, subtraction, multiplication, division, evolution, involution, and the taking of logarithms, applied to complex quantities in any of their several forms, produce only other complex quantities. And hence:

The aggregate of all complex quantities—including all reals and imaginaries, both rational and irrational—operating upon each other in all possible ways by the rules of algebra, form a closed group.

If, in an algebra the elements that constitute the subjects of its operations form a closed group when subjected to a complete cycle of such operations, such an algebra may be said to be logically complete.

Now, in the elementary algebra we teach in the public school, involution and the logarithmic process form an essential part, and through them imaginary and complex quantities make their appearance as unavoidable subjects of its operations. They are necessarily elements coördinate with the real quantities of our algebra, which is therefore logically, and as we now agree, also practically, only the fraction of an algebra, if the complex quantity be left out of it. The inability of the earlier algebraists to recognize this fact made it also impossible for them to carry out the algebraic processes of involution and the taking of logarithms to any except real and positive numbers.

Let us glance, for a moment, at what was further necessary in order to complete the foundations of our mathematical superstructure. The early works upon arithmetic and algebra had been little more than mere collections of rules for the solution of problems. Few

principles were explained, none adequately, and this feature of logical incompleteness has remained prominent, with rare exceptions, in all text-books upon algebra up to the present time. By tradition, algebra became a mere mechanical device for turning out practical results, by careless reasoning errors crept into the explanation of its principles, and through incompetent compilers were perpetuated in the form of current literature, and thus, instead of becoming a classic like the geometry handed down to us from the Greeks in the form of Euclid's elements, algebra became a collection of processes practically exemplified and of principles inadequately explained.

The binomial theorem was either assumed to be true for negative and fractional indices by mere analogy from one or two special cases, or a logically unsound proof was adduced and actually remains in nearly all of our text-books to-day. No attempt at all was made to justify the associative, commutative, and distributive laws in the four fundamental processes.

Hence the necessity of a thorough overhauling of our algebraic system at the beginning of the nineteenth century. We have now accomplished the task; how well, posterity will decide. Abel, in 1829, examined thoroughly and laid down once for all time the conditions under which development by the binomial theorem is possible: and in 1870–71 Weierstrass and G. Kantor gave us a new definition of irrational number, and established the doctrine of the irrational upon a strictly logical basis; and at about the same time (1870), Benjamin Peirce produced the final scientific formula into which our present definition of algebra is cast. It would take me too far afield to go over these matters here in detail. They require, in fact, days,

rather than hours, of careful study for an adequate understanding of them, and I must content myself with referring you to the literature upon this part of the subject under discussion, which is now in existence and easily accessible.

The hurried survey we have now made of the course which mathematical thought has taken within the two great divisions of its work since the earliest times, shows how at every step in the progress of mathematical science, whether in the domain of algebra or geometry, it has seemed impossible for the mind to free itself from the tendencies, or avoid following out the line of development, which some previous age or generation has predetermined for it. Slow indeed has been our progress, if we merely count the centuries through which the struggle against difficulties has been maintained, though the achievement has undoubtedly been very great.

But standing, as we do now, near the close of the twenty-fifth century since mathematics became a science, and taking advantage of the discoveries of our predecessors, we may pass across the entire field that outlines the foundations of algebraic science, and seizing only upon those principles that mark its epochs of advance and are cardinal, construct an algebraic system which shall have all the logical rigor and completeness of Greek geometry; and for the accomplishment of this task no serious difficulty longer stands in our way. The materials are at hand for the purpose; it only requires some master's hand to mold them into coherence.

Let me indicate, in the briefest possible way, what the cardinal principles referred to are, and how they constitute the foundations of algebra. They are the following:

1. The doctrine of proportion, first formulated by Pythagoras (560 B. C.), but systematized and completed by Euclid (300 B. C.). This includes the theory of the irrational, and leads directly to the definitions of multiplication and division in algebra.

2. The introduction of general principles or rules for the fundamental processes of arithmetic and algebra, by the Indians in the early centuries of the Christian era.

3. The introduction of letters and symbols of operation, such as $+$, $-$, $\times$, $\div$, $=$, log., exponents, etc. This has been done gradually, by many hands.

4. The invention of logarithms, by Napier (1614), out of which the law of indices (exponents) also proceeds.

5. The graphic representation of algebraic functions (by curves), by Descartes (1637).

6. The introduction of the continuous variable, by Newton (1665).

7. The introduction and graphic representation of imaginary quantities, by Argand (1806).

8. The final definition of algebra, due to the general reconstruction of algebra during the nineteenth century, by Grassmann, Cayley, Sylvester, Peirce (1845–1870).

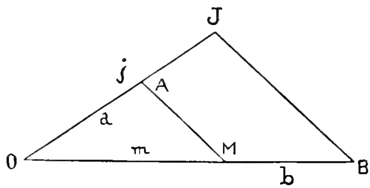
To fill in this outline would be the writing of a book. Two demonstrations must serve the purpose of giving you a hint of the mode of procedure. They shall be the answers to the two questions:

(a) How is the definition of multiplication derived from the theory of proportion?

(b) How, by proper definition and by the help of the graphic method and the continuous variable, can we prove the law of indices (exponents)?

(a) Multiplication.—The propositions of proportion

are assumed to be true, as proved by Euclid, not by the algebraic method used in American text-books. Let a and b be any two magnitudes laid off upon two straight lines, making any convenient angle with each other. At O lay off OB$=b$, OA$=a$, and on OA, in the same direction as OA, lay off OJ, which shall be of fixed length in all constructions belonging to algebra, and shall be called the unit. Join J and B, and draw from A a straight line parallel to JB to intersect OB in M.



Then, by the theory of proportion in the similar triangles OJB, OAM,

$j:b::a:m$. [m=OM].

The line-segment OM$=m$ is defined as the algebraic product of a by b, and is denoted by $a\times b$.*

The law of commutation, namely: $a\times b=b\times a$, follows at once from Euclid, V. 16, which proves that if

$$j:b::a:m$$
$$\therefore\ j:a::b:m,$$

of which proportions the first states by our definition that

$$m=a\times b;$$

the second that

$$m=b\times a.$$

If m be equal to j, then

$$j:b::a:j$$

and b, a are called the reciprocals of a, b, respectively; and they are written thus:

$$j/a=\text{reciprocal of } a,$$
$$j/b=\text{reciprocal of } b, \text{ etc.}$$

* Descartes, *la Geometrie* (1637), reprint of 1886, p. 2.

If $a=j$ in the proportion $j:b::a:m$, then

$$j:b::j:m, \text{ that is } m=b;$$

but by definition $m=j\times b$

$$\therefore\ j\times b=b$$

and in particular

$$j\times j=j,$$

which is the well-known property of unity.

Again, the proportion $j:b::a:j$ defines

$$b=j/a,\ a=j/b,\ a\times b=j;$$

hence,

$$a\times b=a\times j/a=j\times a/a=a/a=j,$$

and similarly

$$b\times a=b/b=j.$$

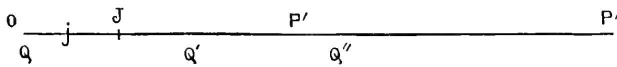
Also,

$$a\times j/b=j\times a/b=a/b.$$

Thus a/b is a fraction or quotient in the algebraic sense.

These examples suffice, no doubt, to convince you that all the laws of algebraic multiplication and division can be deduced as direct consequences from the theory of proportion as laid down by Euclid.

(b) The Index Law.—Suppose P, Q to be two points moving in a straight line, the former with a velocity proportional to its distance from a fixed origin O, the



latter with a constant velocity. Let x represent the variable distance of P, y that of Q from the origin, and let λ be the velocity of P when $x=1$, μ the constant velocity of Q. The velocities that P will have acquired, as it arrives at the positions J, P′, P″, respectively, are

$$\lambda,\ x'\lambda,\ x''\lambda,$$

and the corresponding values of x and y are

$$x=OJ=1,\ x'=1+JP',\ x''=1+JP'',$$
$$y=O,\qquad y'=OQ',\qquad y''=OQ'',$$

Q being supposed to pass the origin at the instant when P passes J, at which point $x=OJ=1$. The velocity of P, relatively to that of Q, is known, or vice versa, as soon as the ratio of μ to λ is given. By means of this construction the terms modulus, base, exponential, and logarithm are defined as follows:

(i) μ/λ, a given value of which, say m, determines a system of corresponding distances x', x'', . . . and y', y'', . . . is called the modulus of the system.

(ii) The modulus having been assigned, the value of x, corresponding to $y=OJ=1$, is determined as a fixed magnitude, and is called the base of the system. We may denote it by b.

(iii) x, x', x'' . . . are called the exponentials of y, y', y'', . . . respectively, with reference to the base b, and the relation between x and y is written

$$x=b^y.$$

(iv) y, y', y'' . . . are called the logarithms of x, x', x'', . . . respectively, with respect to the modulus m, and the symbolic definition is

$$y=\log^m x.$$

Now, by virtue of definition (iii),

$$b^{y'}=1+JP',\quad b^{y''}=1+JP''$$
$$\therefore\ b^{y''}/b^{y'}=OP''/OP'$$
$$=1+P'P''/OP'.$$

But the magnitude $1+P'P''/OP'$ represents the distance that P would traverse, starting at unit's distance to the left of P', on the supposition that its velocity at P' is $\lambda x'/x'=\lambda$, and the corresponding distance passed over by Q is $y''-y'$; hence by definition of the exponential,

$$1+P'P''/OP'=b^{y''-y'},$$

and therefore

$$b^{y''} / b^{y'} = b^{y''-y'},$$

which is the index law for a quotient. In particular, if $y''=0$,

$$b^{0} / b^{y'} = b^{0-y'}$$

or

$$1 / b^{y'} = b^{-y'},$$

and therefore writing $-y'=y$, $b^{y''} / b^{y'} = b^{y''-y'}$ becomes

$$b^{y''} \times b^{y} = b^{y''+y},$$

which is the index law for a product.

The addition theorem for logarithms, namely:

$$\log^{(m)} (y'' \times y') = \log^{(m)} y'' + \log^{(m} y',$$

may be proved in a similar manner from the figures, or it may be deduced as a consequence of the index law, and you will doubtless be willing to take my word for it that the law of involution

$$(b^{h})^{k} = (b^{k})^{h} = b^{hk},$$

together with its inverse logarithmic process

$$h \log x^{k} = k \log x^{h} = hk \log x,$$

can be justified by the methods here described with equal facility.

You will doubtless be interested to know that Napier's original definition of a logarithm was in its essentials the same as that I have just given.

I must not tax your patience longer. If the thoughts I have chosen to lay before you this morning have not touched at many points the work that goes on in your school-rooms from day to day, I console myself with the hope that in your own intellectual life outside the mathematical recitation-room some place may be found for the cultivation of your chosen subject as a science; for a thorough examination of its foundations, for a careful study of its history, and of the life-work of the men who have made it what it is.

I venture to predict that a judicious course of reading in mathematics would have many beneficial effects in your teaching. An anecdote concerning the man who first propounded the *pons asinorum*, or who discovered the binomial theorem, would be caught up by your pupils with eager interest. The great discoveries in science are comparable with the great crises in political history. By what principle in education do we teach our children to revere the names of Leonidas and Warren, because they were the heroes of Thermopylæ and Bunker Hill, but neglect to tell them to whom they are indebted for the knowledge made accessible to them in their arithmetic, their geometry, and their algebra? I recommend as subjects worthy of study and discussion in your class-rooms, sketches in the history of mathematics.

PHYSICS IN SECONDARY SCHOOLS.

SOME ASPECTS OF THE PRESENT SITUATION.

By Frederick Slate,
Professor of Physics.

In appearing before this gathering to-day, it is pleasant to bring with me the thought that conditions as regards high school work have become so much more favorable during the five years that have slipped by since I first made my appearance in this rôle; and in discussing physics-teaching, advocated the policy of making a permanent acquisition of our conceded share in secondary education through reform of method, and clear view of relation to other parts in the scheme of training. Such ameliorations as the present status shows are due, I am glad to be able to say, largely to continuous process, and not altogether to the stimulus which the acts passed by the last Legislature applied.

In the annual tour through the State, which has come to be an important part of my regular duty, the gaps between one high school and the next in which good work in all branches is turned out, are found to be closing up with every passing year. Vivid interest for the best ideas is keeping the workers astir in centers now numbering ten for one that was to be found five years ago; and physics has not lagged unduly behind the general front of the advance.

The University of California made an early beginning, as compared with other institutions of like grade,

in laying down a requirement in physics for matriculation. This subject was introduced as alternative with chemistry or botany nearly ten years ago. In 1887 free option among the group ceased, and since then physics has been exacted of all except matriculants upon the classical and literary requirements. During these later years, too, the examination-papers set, and what other influences could be exerted, were all consistently directed towards securing thoroughness of work, correctness of method, and wise selection of material. There is strong satisfaction in recognizing that the ideas of this kind which our State University championed from the outset, have, at this date, found general acceptance. "Where a concession is made to those who demand some science, * * * physics is the subject selected, because it includes more generic principles than any other science." This quoted sentence carries the authority of the committee report upon secondary education to the next meeting of the National Educational Association at Saratoga, in July, 1892.

While, then, there is cause for self-congratulation; I mean with the secondary education of the State, not with the State University only; it is nevertheless not to be lost sight of that the only mental attitude consistent with progress is that of coöperative inquiry and experiment, in order to sift out what may be regarded as established among the ideas and processes which have been tentatively introduced into school-work; to recognize and adopt them, and thus, while we assure our foothold upon ascertained truth, to narrow down the field within which opinion needs to waver among uncertainties. It is equally inexpedient to look upon our problem as fully and permanently solved, or to think of ourselves as still merely upon the threshold of it.

It may be profitable, therefore, as well as encouraging, if we here put down in brief summary some items in regard to which our position to-day is distinctly an advance as compared with that which we were compelled to occupy only a very few years ago.

First, then, we need no longer to be clamorous about the importance of natural science in general or physics in particular. For causes apparent to most of us, some of which will be alluded to in another connection presently, several features of this work as done in schools have made such strong appeal to popular appreciation, that the various authorities have been easily led to grant freely what has been so unanimously demanded. There was a time when I was obliged to plead for physics; but our present effort can be turned to making wise use of that which is the fruit of old-time exertions.

In the second place, the broader features of the methods most appropriate to the study have been stated and generally agreed upon. If some older ones of us call up in memory the days when we studied natural philosophy as we learned our catechism—by question and answer laid down, and without a vestige of direct reference to the realities to which these statements applied—such memories but swell the tide of humorous pity for the methods in which the earlier generations were schooled; methods so perverse, according to all psychology and wider view of life, that we are reduced to wonder how so excellent a body of men and women could have been shaped in so bad a mold.

And again, the benefits of what we rightly call reform in secondary education have accrued to our branch of study as well as others. We have cut down the number of subjects taken up, and devoted a larger fraction

of the pupil's time to each; nor is there reasonable doubt that we shall find advantage in continuing this process beyond any point yet attained. In so doing we gain opportunity to impart what I call serious knowledge; to develop each branch to the degree necessary for making permanent impression upon young brains; and to lift ourselves out of the slough of superficialness into which high schools have been brought by a course of study touching upon almost all sides of learning.

Indirectly, too, we are helped by the demand this thoroughness in what is taught makes upon the one who teaches. In a course laid down according to modern ideas, there is no place for any except those well-trained and fully-prepared to teach sound knowledge on the lines of good methods.

With place assured, plans well outlined, and an emphatic demand for excellence of instruction, we have within reach good leverage for further progress; and we must not shrink from the weighty responsibility of keeping our "intellectual conscience" awake in its search for still better things, and of conforming our practice to them as soon as found.

What I have further to say is to be thought of as my endeavor to formulate for myself and for you the truth about what are now either (1) unclearly-held views, or (2) false tendencies, needing to be combatted, or (3) elements worthy of being included in our teaching.

It is altogether likely, indeed, that some of you have heard from me, on various occasions and disconnectedly, all that the allotted time allows me to bring forward here. Nevertheless, the more public utterance and discriminating statement seem to offer advantages which warrant what may be introduced of repetition.

For some years past, we may even say ever since its introduction as a feature of school-work, the laboratory has offered themes fruitful of thought, and fruitful of discussion because of divergence in thought. It will be a distinct gain if we may contribute aught to-day towards fixing the scope and limitations of laboratory work, so that we are able to say of its field of usefulness: "It lies thus and so, and is hedged by these plain boundaries." If we can in any fair measure accomplish this, there will follow clearer insight into the reason for its existence as a feature of science-teaching at the elementary stage; better judgment as to its necessity or indispensableness.

It would not surprise me greatly to discover a number present holding a discussion of these points to be a veritable threshing of chaff; but the very diversity of opinion and practice proves them undecided now, nor would it be anything unique if we should find that they had never been clearly thought out by many of those who zealously take part in introducing the laboratory system. For it is rather characteristic of the early stages in reform movements, that the rank and file at least are, in the literal sense of the word, adherents, and cleave to the view which happens their way, while the privilege of holding conviction as the result of thinking, and at the same time making it the basis of action, is reserved to the few who are akin to the generations following that in which their lot is cast.

It would be to some extent true, I am sure, if we should trace a partly imitative process, which found the University laboratory in existence with its function of research, modeled the college laboratory after it, and copied this feature into the school system, without pausing to very clearly define how the original function

must be modified to suit the changed conditions. This is not said at random; I have been met on many occasions with distinct indications of this sequence.

Now it is apparent that out of the school-laboratory comes no investigation, in the sense of "increase of ascertained truth." Rediscovery of truth by the pupil there may be; ought certainly to be, and to a large extent, we find maintained, and we shall revert to this before leaving the topic; but the function of the matured scientific worker does not yet come into action.

In another direction there are signs of attempted alliance with entirely different tendencies. About three years ago the idea of manual training received a strong impetus; some projects were even entertained and discussed for engrafting it upon the common school system. Did we not note movements of attraction between laboratory and manual training shop? It would be wrong to deny the existence of common ground on which these two different ideas overlap. But coalescence would not have been in the true interest of either, and we must all have been glad to see the danger fade away.

I spoke freely on this subject at the Sacramento meeting in 1888; if I make allusion to it here, it is for the purpose of illustrating how a thing may *be*, and yet those who are called upon to administer it be wrapped in mistiness as to its true mission. The well-balanced mind avoids confounding these matters, as the eye escapes the confusions of color-blindness; by having within it faculty for perceiving all primary elements, and distinguishing shades and tints as blendings of these in varied proportion.

Well, if the essential of laboratory work in schools be not manual training, that is, apparatus building, and

cannot be discovery of new truth, may it not perhaps be new discovery of truth? Shall the classes be so conducted that the pupils are—to put the extreme case—told to make an experiment, being left uninformed how the result should fall out? I could quote very respectable authority for the criticism upon certain books: "Their scheme is all wrong, because they tell the pupil beforehand what to expect." That the proposed plan quickens observation, cultivates the investigating habit of mind, and lends keen zest to old experiments, although the sweetness of discovery is but tasted with the tip of the tongue, and is, after all, only of the kind we see in our boys as they hang breathless over Ivanhoe or Robinson Crusoe—all this must be readily conceded by every candid mind. There is much that appeals to me in an inclusive view of education as a thinking again of the thoughts of the race; with many foreshortenings, to be sure; with extensive elimination of periods of groping, and much redistribution of emphasis; but in the main a bringing into contact with the recorded thought of humanity. So, too, there is of necessity, under circumstances as sketched above, much of that individual treatment for pupils which is a marked characteristic of best education; to be striven after at every point among the elements opposed to it which the handling of crowded classes entails.

If we tone down the leading idea from rank extreme to somewhat of moderation, I can fancy such a charming picture of wise teacher in the midst of alert pupils, each actively thinking for himself, and aided to the extent and in the way suggested by the obstacles that block his path, that I am willing to let this ideal of laboratory stand bright before us, undimmed by any

critical analysis. Pupils *may* be led from stepping-stone to stepping-stone with proper, half-dissimulated guidance, avoiding treacherous footholds, and thus springing nimbly and securely over many a gap where the path-finder stumbled and fell short.

Let our further consideration and argument, therefore, be confined to examining what results may be gained by executing a plan like this, and seeing in how far, as well as in what directions, such procedure needs supplementing before the requirements of the case are fully met. An examination of this sort cannot but prove fruitful if its brings out the truth, unconsciously possessed it may be, which gives to our plans and ideas their real value—value often ascribed to widely different sources.

The word "rediscovery" seems to me apt as suggesting a feature in the working of the pupil's mind akin to what we find historically true in the process of original discovery. I mean that the results obtained involve a large element of intuition, and are not conclusive and unassailable, as they may become through later scrutiny and test. It is strikingly shown, as we follow the course of development in the work of men like Galileo, like Huyghens, like Mayer, even, how incompletely established certain most valuable results were at the date of their first announcement. As in Alpine climbing the leader on the rope is often obliged to content himself with precarious foothold and slippery grasp as he picks the way, while those behind him secure themselves in notches cut, so Newton's minute examination for accuracy is needed to confirm Galileo's results; Young's labors bring testimony in support of Huyghens's generalizations, and only through Joule's patient detail comparisons is full acceptance gained for Mayer's ideas.

At the very best our pupils repeat upon a minute scale the experiences of the former group, not those of the names in the second.

We hear much said of the "inductive method," which is often understood in a way that may be characterized as pretending—with more or less truth admixed—that we do *not* know; but, as a matter of fact, hypothesis generally precedes experiment, and we test in order to discover whether we *do* know. Jevons has clearly, and as I think justly, pointed out how essential to the real process is this rapid flashing back and forth between assumption and putting it to the proof.

Have we not all in mind Faraday's form of stating the truth? "You must tell me what I am expected to see," he said, "before I can know whether I see it or not." That is, the mind must be prepared by hypothesis; we must look at phenomena, not with dispersed bovine gaze, but with retinal-spot highly sensitized for the one thing among the mass which are in sight. Ability to prophesy is the criterion of true knowledge. The bearing of these remarks will be plain, I hope, when we remember that our young people are doing their work under similar conditions, and in addition are limited by their feebler powers.

We must not expect, then, that laboratory practice, even for a favored group, is going to emancipate from all reliance upon authority, and enable us to write as device: "Accept only what you have yourself worked out;" or even the modified form, "What you have yourself thought out." By occupying such extreme positions, which are in reality untenable, we constrain ourselves into adopting for defense controversial methods, and losing that candor of mind which lies open to the truth.

In the light of many years spent in teaching and thinking, it seems impossible to affirm that, try as we may, there can be eliminated from our teaching the element of acceptance upon authority. Beginners must continually take bearings as they advance, and find whether they are in the beaten path, as I am told the geological survey runs its traverse-lines with repeated adjustment to the previously established points of an accurate triangulation. And ought this element to be cast out root and branch as wholly bad? Is not the claim that it should based upon extreme reaction against preponderant dogmatism? It is true, rather, that a part of all sound education is to cultivate due reverence for the past, and esteem for the legacies handed on to us from its worthies. The great and the good have worked to little purpose, if we are to refuse their inheritance unless every coin of it has been tried upon each individual touchstone. Nor should we fail to see the artificialness of a system which sets up, within a small range and during a limited period, standards whose use cannot be extended in time or space to cover human life. It is true of mature men of widest power that they command with independent research a small arc only upon the circumference of knowledge.

In entering this plea for a remainder of authoritative teaching. I am running a-tilt against no mere man of straw. It is unfortunately true that just such extravagant claims as are here implied are made for the kind of work that we are discussing, which tend to bring it into disrepute with heads clear enough to see the fallacy of the position assumed. Interest in the adoption of the best methods demands that we should avoid such extremes as have been held up for criticism. We accept the law of gravitation, or the law of reflection for light,

or Ohm's law for electrical currents; in fact the great body of laws which constitute what may be called formal physics—laboratory-work of school grade can at most think consequences of these and try to realize them. As a recent writer has said (and I think his utterances are the best I have seen in print upon the subject, perhaps because they come in as independent corroboration of what I have long held and preached): *"The laws and principles which have been most carefully studied by scientific men should be made the instruments, not the objects, of scientific research. The teacher should avoid, as far as possible, experiments whose ostensible object is to establish well-known facts, like the law of conservation of energy, the truth of which is not really in question. But the use of the experimental method, as an illustration of such laws, is not to be denied. It is only through the aid of definite examples that most persons can arrive at an understanding of physics."

A trenchant blow is struck in these sentences, every phrase of which counts, against the central fallacy of much that is carried out in school-work as quantitative measurement. It cannot be regarded as proving the law; but it is a proving of the pupil; of his conscientiousness, definiteness of knowledge, and power to take pains. Here lies, indeed, the connecting link between manual training and laboratory work, especially of the quantitative kind. Curiously enough this bond attaches on what we may denominate the ethical side. The constructive use of principles or materials has a reactive high moral value when wrong figures, or errors cut out in wood or iron, bring us face to face with faults in knowing or executing. A shrewd friend said

* Whiting, Physical Measurement, p. 599.

to me many years ago, that, during the period when we are fostering the thinking faculty in the younger generation, those subjects are the best adapted tools, which give the most frequent and accessible check and control upon the results of their essays at thinking. This is the kernel of the truth, I am sure. It may be useful to the unification of our thought to see a large share of the educational value in such subjects as geometry lying within these lines. There we have flawless standards, it is true, in hypothetical assumptions, whose discord or harmony with conclusion gives certain answer as to correctness of reasoning; but laws of physics involved in school work are scarcely behind geometrical axioms in practical freedom from doubt. I am mistaken if some of the readiness with which physics has been incorporated into our schemes of study is not due to the recognition that it furnishes a large and new available supply of such material.

It seems true, in spite of the claim that science-work in schools is inductive (meaning exclusively so), that we must make allowance for the existence of a considerable element which offers the essential feature of deduction. Namely, that certain ideas are assumed; with a coloring of reason offered, to be sure, but necessarily without critical analysis, weighing, and discussion; and of these the pupil thinks a corollary and works it out experimentally. As far as this kind of mental exercise is concerned, there is little to choose between laboratory problem and one set in geometry.

This connection leaves the door open to place a word in defense of qualitative experiments, which it is the present tendency to reject almost entirely. But the intuitive activity of the mind finds exercise chiefly in qualitative work. It is here that rediscovery, with its

(rather fictitious) novelty, can be properly spoken of. I know that students in my own classes often show the onesidedness that is the fault of familiarity with those experiments exclusively which are employed because they afford exact methods, while experiments are passed by that yield only views of connection between cause and effect in phenomena. In the school-years such exclusiveness is still less in place, so that this thought deserves of us that we bear it in mind and correct our practice by it.

On gathering together what these considerations have thus far shown, we shall have before us the answer, as I conceive it, to the question which concerns itself with results to be expected of our ideal laboratory. Those who are subjected to its discipline may be guided to infer the qualitative statement of physical laws; they may experimentally learn to know what are the main determining causes of a wide range of phenomena. Their conclusions will be compared with established truth mainly, and their correct understanding of relations will be brought out by measurements which involve the application of assumed quantitative laws. As bye-products they will have gained a fair degree of manipulative skill—manual dexterity, that is; and, if it be fair to call this a bye-effect, a set towards conscientious accuracy and the facing of facts that ramifies widely into the fibers of character. And all of this with healthful stimulus of interest and development of faculty upon the two great lines of inductive and deductive reasoning.

We may be content to call this a goodly harvest, although our claims are modest by the side of those I have heard advanced. Training like this is desirable, and that although the actual profit-sheet may never

show as large dividends as these. Our minds are prepared to meet a coming-short of ideals in any educational scheme as applied to a particular group of pupils; but it is of the highest importance to know whither our effort should tend, and to hold ourselves and our young charges up to aim for the very best.

Another branch of our inquiry raises the issue whether such laboratory training is likely to be sufficient, or whether it needs supplementing. Experience seems to show that it does. I will try to present the matter through an analogy. The suggestion that comes from several parallel cases is of the same tenor, and we should heed it. Time was, when the study of the classics was prepared for by solid digging at the grammar, which was mastered by rote before any venture was made in reading the literature. This period was succeeded by a strong reactionary tendency to abolish the study of formal grammar, and to derive all such material from the text. It has been found, however, that the same high level of attainment cannot be reached if the grammar-book be dispensed with. It is introduced at a different stage, in less isolated fashion, with larger proportion of text-reading, and truer exposition of the principles of grammar, as inferred from the observed phenomena of the language used; but no substitute completely replaces it.

I hope for the day when the school-grammar of physics shall be wisely written, fully mastered in connection with our reading of the text, which is Nature, with proper understanding of the statement of formal law as general truth obtained by collation of phenomena. We have had our days of rote-learned grammar; we have had our phase of undue reliance upon text-reading and consequent slovenliness. Perhaps better things lie just ahead.

There is at the present time a well-marked drift towards removing difficulty from the pathway to knowledge, which gives a kind of general support to the belief which has spread abroad in regard to laboratory methods, that they make subjects entertaining to the degree of being learned as play. There should be rejoicing over the disappearance of those difficulties which have their source in allowing the end to be eclipsed behind the means to the end, in perverse routine, and the whole apparatus of pedantry; that evil spirit whom we should all be glad to know of as exorcised from our school-rooms forever. But clearing away this rubbish does no more than render it possible to concentrate effort upon the real inherent resistance of the subject, as the felling of a tree is facilitated by removing the brush which deadens the axe-blows. Difficulty must continue to meet those who attack whatever possesses sterling educational value; it is, in fact, one stamp in the hall-mark.

I do most sincerely maintain that we should put work before our young people which requires effort, strenuous effort, in order to accomplish the tasks set. We must continue to place a high value upon the years of adolescence; the period of rapid expansive growth and strong assimilating power for mind as well as body; the time during which the habit of consecutive effort should become ingrained. We can do no better service to pupils nearing the end of their high school course, than to claim and gain for them leisure to satisfy the requirement of thinking, and growing strong in controlled power to think, while we discourage nervous cram and sham acquisition.

If, then, to turn from general statement to the matter in hand, there are, as I have tried to show, needs for

systematic knowledge of principles, rooted in the subject-matter; existing, not as alternative to laboratory-work, but as condition that an important part thereof may be successfully prosecuted; it is acting in the interest of the pupil if we do not shirk the task thus imposed, of encountering a citadel of difficulty transcending those met in the recreative experimental outworks. We are thus helping to inculcate the lesson which is the quintessence of so many lessons; that is, in Huxley's words: "The most valuable result of all education is the ability to make yourself do the thing you have to do, when it ought to be done, whether you like it or not. * * * However early a man's training begins, it is probably the last lesson that he learns thoroughly."

Our newest text-books do not help us as they might just here. Their abstruseness has been largely toned down by the wholesome and clarifying influences of experimental exercises. So much thought is excited by what is taken in hand, and so much explanation called for of every-day occurrences, that the elimination of advanced theories may be relied upon to work almost automatically. We can, if necessary, aid this process by judicious omissions; but the lax incoherency of some of our books is not so easily neutralized.

We are all aware that text-books are by no means mirrors reflecting opinion current among experienced teachers; so it would be unfair to judge practice by them. All teachers, and the best teachers most of all, make discriminating use of their books, to the extent often of recasting the matter in presentation. But print is powerful, and not yet altogether divested of sacredness; the molding influence of the book, too, is felt most strongly by those very younger members of the profession upon whose development it is most

urgent to impress the right direction. It seems permissible, therefore, that I should devote to criticism of our text-books a few further words, which are a protest against the lack of proportion in their treatment of topics; and in saying this I have electricity quite prominently before my mind. It may look paradoxical to raise outcry against magnifying those applications of physics which have brought an influx of students—20 for 1 fifteen years ago—and have unlocked coffers for the building of laboratories, besides forming the "crowning success of the century," as they have harnessed a new form of energy in the service of man. Such plea comes with all the more force and temptation, because the time in which we are sits dazzled under the sway of electricity. We no longer realize perspective in these things; but physics is in danger of being swamped by the unexampled material success in this one of its divisions, as an examination of our colleges easily shows. And so in the schools, possession is taken of our books from the early pages, in which centimeter-gram-second system is forced upon pupils unable to comprehend it, to the Volt, Ampere, Wheatstone bridge, and other members of the goodly company that bring up the rear.

Application of principle to cases occurring in daily life meets with my strongest approval. I can say, in the simple words of Maxwell:

* "Science appears to us with a very different aspect after we have found out that it is not in the lecture-room only, and by means of the electric light projected on a screen, that we may witness physical phenomena, but that we may find illustrations of the highest doctrines of science in games and gymnastics, in traveling

* Scientific papers, Vol. II, p. 243.

by land and by water, in storms of the air and of the sea, and wherever there is matter in motion.

"This habit of recognizing principles amid the endless variety of their action can never degrade our sense of the sublimity of nature, nor mar our enjoyment of its beauty. On the contrary, it tends to rescue our scientific ideas from that vague condition in which we too often leave them, buried among the other products of a lazy credulity, and raise them into their proper position among the doctrines in which our faith is so assured that we are ready at all times to act upon them."

But when one most modern application is seen overshadowing all others with a predominance due to evanescent causes, I feel we are making a blunder if we yield to the pressure. For physics is the classic among the sciences: it forms the permanent record of man's success in accumulating, through centuries of effort, a reasoned explanation of large groups of phenomena. Instead of reserving chief emphasis for the most modern contributions, and thus ignoring an opportunity, which is so plainly ours, of casting off the reproach urged against science: that it is lacking in the human element; we should aim at bringing out the historical aspect of discoveries as they come up for treatment, beyond the mere uncommented mention of a name, and at exciting lively interest in this picture of international coöperation upon an edifice which has been building through more centuries than Cologne cathedral, and which is more imposing to the mental vision, although less substantial than stone and mortar. I recommend to you sketches from the history of physics as valuable material for the class-room—well adapted to enforce the lessons most desirable to impress;

of painstaking slow advance, and hard work over every inch of conquered ground.

At several points in what precedes, I have implied the close relation among the several branches of high school work by insisting upon the results of comparing them in parallel. Before bringing these remarks to a close, I should like to urge this idea more explicitly, as its value deserves.

The task of the public school on the intellectual side may be considered as being to: (1) Impart useful knowledge; (2) Promote clearness of thought; (3) Cultivate accurate expression. We should find earnest striving after these three things in every class-room. Where this is the case with studies in language, mathematics, and science, each pair buttresses the third, and the unity of purpose sets its stamp upon the pupil's mind.

For reasons connected mainly with the need of imparting varied and extended information, we segregate into separate class-rooms, and place in charge of teachers specially qualified to handle and expound portions of the material which it is advisable to treat. Yet, after all, in our thought and in their essence, these are but fractions of a unity; these comings and goings and modulations of key can be grouped into a harmony of movement and of utterance for one central theme.

Last Spring, an eminent preacher and lecturer developed for me, in conversation, the idea that all our outdoor sports are wrong in principle, because they involve a striving of one, or of one crew, for victory, which must also mean defeat. The truer human end would be reached, he maintained, by including in one effect the efforts of all, combined with proportion and rhythm. There is perhaps too much of the Anglo-Saxon in me

to quite accept this view as to field-sports. I am not convinced that rivalry in contest is not succeeded by generosity in those who excel, ungrudging praise in those who are proved inferior. But within the range of education I can find deep truth in this view, and do regard it as bringing to utterance soundest doctrine.